AERODYNAMICS OF FLOW IN A PLANE DUCT WITH SUDDEN EXPANSION

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The Navier – Stokes equations are solved numerically for laminar viscous incompressible fluid flow in a plane duct with sudden expansion. The solution is compared with experiment.

The flow of a viscous incompressible fluid in a plane duct with a sudden expansion is accompanied by flow separation from the duct wall and the formation of rotational flow zones. Ducts with a sudden expansion are used as devices for flame stabilization and enhancement of heat-transfer efficiency. The aerodynamics of such flows is investigated by three basic approaches:

a) the synthesis of simplified models of flow with separation zones, based on the results of experimental studies at Reynolds numbers $\mathrm{Re} \simeq 10^4$ to 10^5 [1-11];

b) the derivation of precise asymptotic solutions of the Navier – Stokes equations for only large (or only small) Reynolds numbers. This method is used to investigate supersonic flows with laminar and turbulent separation zones of large dimensions in comparison with the thickness of the boundary layer at the separation point [12-17];

c) the development of numerical methods of solution of the boundary-value problem for the Navier – Stokes equations. Surveys of the numerical integration of the Navier – Stokes equations with the application of various finite-difference schemes may be found, along with exhaustive bibliographies, in [18-21].

Simuni [22] has investigated viscous incompressible flow in a duct with a sudden expansion and constriction during motion of one of its walls, up to numbers Re = 1000. The scheme used in [22] is rendered unstable by the symmetric approximation of convection terms. A numerical integration of the Navier – Stokes equations is carried out in [23] for the problem of the evolution of viscous incompressible flow in an axisymmetric tube with a sudden expansion.

The system of Navier – Stokes equations describing plane flow of a viscous incompressible fluid is written in the dimensionless form [24]

$$\begin{gathered}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),\\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right),\\ \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.\end{gathered} \tag{1}$$

We use the stream function ψ and vorticity function ξ,

$$u=\partial\psi/\partial y,\quad v=-\partial\psi/\partial x;\quad \xi=\nabla^2\psi. \tag{2}$$

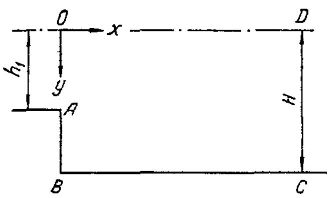

Fig. 1. Flow diagram.

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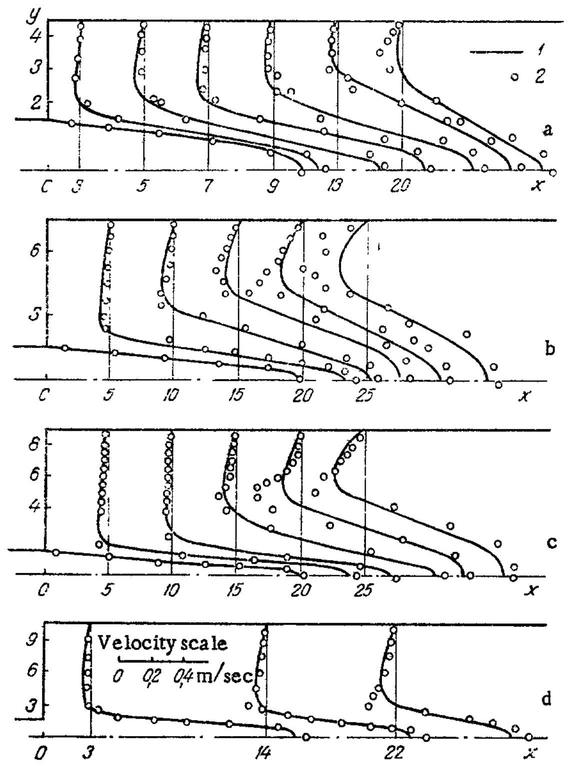

Fig. 2. Velocity profiles in a plane duct with sudden expansion. 1) Calculated by investigated scheme; 2) experimental data; a) $H/h_1 = 3$; b) 5; c) 6; d) 7.

to transform to a system of two equations (ψ, ξ), which we write in divergence form [25]:

$$\frac{\partial \xi}{\partial t} = \frac{1}{\mathrm{Re}} \nabla^2 \xi - \frac{\partial}{\partial x} (\xi u) - \frac{\partial}{\partial y} (\xi v), \quad \nabla^2 \psi = \xi. \tag{3}$$

Then in our case the Gauss divergence theorem is satisfied identically by equations in finite-difference form, independently of the solution precision [26].

In solving the system (ψ, ξ) (3) it is necessary to state the boundary conditions for ξ. These conditions can be deduced approximately by means of the values already obtained for ψ. Various methods of deducing the boundary conditions for ξ are discussed in [27].

In the present paper we investigate an implicit second-order two-layer difference scheme with the one-sided approximation of convection terms. We represent the diffusion terms by means of ordinary central differences [18], and convection terms by the method of comparison of differences in the upstream direction at three points, since we use approximation with error $O(h^2)$. In the sense of convective transfer information is transmitted into a cell only from upstream cells. When the sign of the velocity changes near a corner point, it is necessary to modify the main difference-comparison scheme in the upstream direction [28]. At a distance of one space step from the boundary of the region for convective terms we use approximation by central differences. Thus, the finite-difference system approximates the system (ψ, ξ) with error $O(h^2 + \Delta t)$.

Stability analysis of the investigated finite-difference (ψ, ξ) system is carried out by the Neumann local-linearization method [19]. A sufficient condition for stability, including the influence of the boundaries, can sometimes be obtained by energy analysis [19], but this approach is difficult even for simple differential equations. For the scheme discussed above we have taken the stability condition in the form $\Delta t \leq a h^2$, where a is a constant of order 1/2.

Let us consider the initial and boundary conditions for flow in a suddenly expanding plane duct (Fig. 1):

$$\psi = 1, \quad \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \psi}{\partial x} = 0 \quad \text{at} \quad ABC,$$

$$\psi = 0, \quad \frac{\partial\psi}{\partial x} = 0, \quad \frac{\partial^2\psi}{\partial y^2} = 0 \quad \text{at } OD,$$
$$\psi = \int_0^y f(y)\,dy \quad \text{at } OA,$$
$$\xi = 0 \quad \text{at } OD; \quad \frac{\partial\psi}{\partial x} = 0, \quad \frac{\partial\xi}{\partial x} = 0 \quad \text{at } DC, \tag{4}$$
$$\xi = \frac{8\psi_{w+1} - \psi_{w+2} - 7\psi_w}{2h^2} \quad \text{at } ABC,$$
$$\xi = \frac{2(\psi_{w+1} - \psi_w)}{h^2} + f'(y) \quad \text{at } OA,$$

where w designates a boundary point; w+1, w+2, ... designate the internal boundary points closest to w; and f(y) is the known velocity distribution at the entrance, which is obtained from experiment. We note that the conditions on the line DC (4) correspond to the condition of equalization of streamlines far downstream. In the computation the line DC is situated a finite distance from the initial cross section. To satisfy the conditions on DC we use a nonuniform grid with respect to x so as not to increase the number of nodes and "transport" the boundary conditions too far downstream [30]. In [31, 32] the Navier – Stokes equations are closed downstream by equations of the boundary-layer type. In [25] linear interpolation of the stream function and vorticity inside the flow region is used on the line DC.

The resulting system of finite-difference equations is solved as follows. Let the grid functions $\psi^n_{i,j}$, $\xi^n_{i,j}$ be known at a certain time. From the difference analog of the first equation of the system (3) we determine $\xi^{n+1}_{i,j}$ at interior points of the grid region. Solving the difference analog of the second equation of (3) by the iterative procedure of Seidel for known boundary values of $\psi^n_{i,j}$ (4), we find $\psi^{n+1}_{i,j}$ inside the region [29]. From the value of $\psi^{n+1}_{i,j}$ we determine the vortex $\xi^{n+1}_{i,j}$ at the boundary. The computation was carried out on an M-220M computer. The difference grid has a maximum of 1500 nodes. The grid spacing is $\Delta x = \Delta y = h = 0.1$. For a certain time value the output data of the program are as follows: velocity profiles u and v in the duct cross sections; values of the coordinate y of the streamlines $\psi = \psi_i = \text{const}$ at $x = x_i$. The fields of the stream function ψ and vorticity ξ are printed out at the end of the computation.

To corroborate the numerical results we conducted experiments on laminar viscous incompressible fluid flow in a plane duct with a sudden expansion. The velocities in the plane duct with a sudden expansion were measured with a Disa Elektronik (Denmark) hot-wire instrument consisting of two type-55A01 hot-wire anemometers and a 55A06 correlator (diameter of sensing wire 5μ; error at low velocities – 4 m/sec or less – 0.2%). The velocity field was measured in cross sections of the duct with expansion ratio $H/h_1 = 3.0$, 5.0, 6.0, 7.0 (Fig. 2). The results indicate that the velocity field at the slot exit can be described by the following equation for laminar flow in a plane duct:

$$\frac{u}{u_m} = 1 - 4\,\frac{y^2}{h_1^2} \tag{5}$$

($u_m = 1.62$ m/sec, $h_1 = 1.5$ mm, $\text{Re} = u_m h_1/\nu = 162$). The experimentally determined velocity distribution at the slot exit differs from the velocity profile computed according to (5) by a maximum of 1%.

In Fig. 2 the computed velocity distributions of the duct flow are compared with the experimental data. An analysis of the velocity distribution for various expansion ratios shows that the length of the recirculation zone and the velocity in it depend strongly on the expansion ratio; for example, for $H/h_1 = 3.0$ the maximum velocity in the recirculation zone is about 10% u_m, while for $H/h_1 = 6.0$ it jumps to 20% u_m. The numerical computation indicates more rapid decay of the axial velocity than the experimental (Fig. 2). Inasmuch as we have transported the conditions at infinity to a finite distance from the expansion step, the conditions specified at the right boundary were extremely "rigid." Consequently, a more rapid decay of the axial velocity is observed. On the whole the comparison of the numerical with the experimental results shows that the given numerical method ensures reasonably accurate results in the computation of such flows.

NOTATION

x, y, dimensionless coordinates referred to the characteristic length L of the duct (Fig. 1); u, v, velocity components along x and y axes, referred to the characteristic velocity u_0; P, dimensionless pressure; $\text{Re} = u_0 L/\nu$, Reynolds number, referred to the characteristic velocity u_0, characteristic length L, and kinematic viscosity ν; i, j, indices numbering grid nodes along x and y axes; n, iteration index; Δx, Δy, grid spacings along x and y axes; Δt, time step; u_m, maximum velocity on duct axis at entrance; h_1, H, half-widths of duct at entry and exit (Fig. 1); ∇^2, Laplace operator.

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ENTRAINED DOWNWARD FLOW OF A GAS AND A LIQUID IN A VERTICAL CHANNEL

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The fluid friction and breakaway of liquid from the surface of a film are investigated and described quantitatively.

Relatively few investigations have been devoted to the interaction of gas and liquid flows in downward and upward entrained motion [1-6]. This type of flow takes place in various heat- and mass-transfer devices and permits appreciable intensification of heat-transfer processes at a solid — liquid interface as well as mass-transfer processes at a liquid — gas interface. The efficiency of heat- and mass-transfer equipment is improved in this case by the reduced film thickness and increased velocity in the film under the action of the gas flow and inception of turbulence in the gas layers adjacent to the film surface, all of which intensify heat and mass transfer at the liquid — gas interface.

The annular-mist flow regime covered by the results of the investigation described below is observed at high velocities and large volume contents of the gaseous phase. It is characterized by two sharply differentiated flow zones: 1) the wall zone, which is occupied by a thin liquid film; 2) the flow core, in which the gaseous phase moves together with suspended droplets of the liquid phase, which are broken away from the surface of the film. The velocity of the film is much lower than that of the gas. Mass transfer takes place between the film and the core, further intensifying the heat- and mass-transfer processes and simultaneously increasing the fluid-friction (viscous) losses.

We have investigated the mass transfer and fluid friction associated with entrained downward flow of a liquid film and gas core on an experimental setup consisting of a working section in the form of a vertical transparent plastic pipe of inside diameter $d = 40$ mm and length $l = 2000$ mm. In the upper part of the working section we installed a film-forming device in the form of an annular injection slot of width 0.6 mm, along with an air-injection device comprising a Vitoshinskii nozzle, which provides a uniform gas-velocity profile in the entry section. The investigations were carried out for a gas — liquid system with water and air under isothermal conditions.

The water entered a supply tank, in which an overflow pipe was installed to maintain a constant level, thus assuring a constant flow rate throughout one experiment. From the supply tank the water flowed into the film-forming device. The static pressure was tapped at distances of 35, 895, and 1915 mm from the entry of water and air into the working section, making it possible to determine the pressure drops along the length of the channel.

A chamber was placed in the lower part of the working section to permit separation of the liquid streaming along the pipe wall as a film and the flow core of air containing suspended detached liquid droplets. The latter were captured by cylindrical sampling probes with diameters $d_i = 10$, 20, 30, and 35 mm, which were mounted at a distance of 2000 mm from the entry to the working section and subsequently delivered their contents into the measurement vessel.

The mass-flow of water in the experiments was $(1.54 \text{ to } 12.56) \cdot 10^{-2}$ kg/sec, corresponding to a variation of the spray density Γ from 0.125 to 1.6 kg/m · sec. The water temperature in this case was 8 to 12°C, and the Reynolds number of the liquid had values

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